Point Process Models Allowing for Interaction and Inhomogeneity

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Data sets consisting of positions in the plane or in space (point patterns) can be modelled using spatial point processes. Often the points show dependence in form of inhibition or clustering (interaction), and in addition a trend in the intensity of the points (inhomogeneity) is often observed. My thesis concerns the study of point process models that can describe interaction as well as inhomogeneity.

The behaviour of a point process is usually measured relative to the homogeneous Poisson point process. In this model the points are independently distributed, and the number of points in an area is proportional to the size of the area. Inhomogeneous Poisson point processes can be used to model point patterns where there is a trend but no interaction. The important class of Markov point processes is very useful for describing interaction. However, statistical tools developed for Markov point process models have mainly concerned homogeneous models.

The first part of the thesis is a review briefly describing four model classes capable of modelling interaction as well as inhomogeneity. The models all build on a homogeneous Markov model which is modified in order to introduce a trend. For all models, the inhomogeneous Poisson point process plays an important role. The most basic way to obtain a trend is to change the reference process from a homogeneous Poisson point process to an inhomogeneous Poisson point process. In the first type of model, the trend is obtained in this way. In the three other model classes, the trend is obtained by position-dependent thinning, transformation and local scaling respectively. These models can also be defined with the inhomogeneous Poisson point process as a reference measure. The resulting point process models are essentially different. The review concludes with an informal comparison of the models, emphasising their differences and individual strengths, and suitable fields of applications are suggested. However, when the interaction is weak or the trend is small, the differences between the four models fade, and they might serve as alternatives of each other.

Two of the model classes considered in the review are transformation models and local scaling models. The main part of my thesis has concerned the development of these two model classes and a study of the statistical properties of the transformation models. The work on transformation models has been performed together with my supervisor, professor Eva B. Vedel Jensen. The local scaling models have been developed together with her, Dr. Ute Hahn (Institute of Mathematics, Augsburg University), and Dr. Marie-Colette van Lieshout (Centre for Mathematics and Computer Science, Amsterdam).

The main contributions to the thesis are 5 papers written for publishing in journals. Three of these are on the transformation model, one is on the local scaling model, and one is a review paper that presents and compares the statistical properties of the transformation models and the two model classes for inhomogeneity and interaction also described in the review.

A transformation model is a model of interaction and inhomogeneity that differs from previously suggested existing models because the interaction is position dependent. In one of the papers we argue that the transformation model makes it possible to estimate the trend in a point pattern without taking the interaction into account. Since the trend is introduced into the model by a transformation, it is possible to transform the observed inhomogeneous point pattern into a homogeneous point pattern. Then well-known analysis tools for homogeneous point patterns can be utilised to describe the interaction.

A technical report is also attached to the thesis. The computational challenges involved in analysing a particular planar point pattern using the transformation model are described in detail.

Since the trend is introduced into the transformation model by deformation of a homogeneous point process, the interaction is of anisotropic nature. Interaction of this type is appealing if the point pattern has been formed by physical deformation, for instance during a growth process or during stretching/compression of the material containing the points. In other situations it is of interest to describe interaction that looks similar in different areas, but on a different scale. This might involve plants whose environmental conditions cause some areas to be sparsely covered and other areas more densely. In dense areas the plants stand closer than in sparse areas, so the point pattern looks locally like a scaled version of a regular homogeneous point pattern. The local scaling models are able to handle interactions of this type. As for the other three models of inhomogeneity and interaction mentioned, local scaling models are also obtained by modifying a homogeneous Markov point process in order to obtain the trend. However, rather than deforming the space where the points live (as done for transformation models), local scaling models are constructed by changing the measures by which distances, areas and volumes are calculated. Thus, the usual volume measures are replaced by locally scaled measures, obtained by using a scaling function. We prove that in areas where the scaling function is constant, local scaling coincides with global scaling. The statistical properties of this model still need to be investigated. However, we have reason to believe that non-parametric kernel estimation of the intensity can be used to estimate the scaling function.